Supporting Information for "Why does tropical convective available potential energy (CAPE) increase with warming?"

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Text S1: Simple zero-buoyancy model

The simple zero-buoyancy model is based on a simplified thermodynamics in which there is no ice phase. In addition, the effect of water on the density and heat capacity of air is neglected. Accordingly, MSE is defined here as $h = c_p T + L q_v + gz$. The plume equations describing the vertical profiles of saturated MSE h^*, total water ($q_t = q_v + q_c$, where q_c is the non-precipitating condensed water), and pressure are:

$$\partial_z h^* = -\epsilon \left(h^* - h \right) \ , \tag{1}$$

$$\partial_z q_t = -\epsilon \left(q_t - q_v \right) \ , \tag{2}$$

$$\partial_z \log p = -g/(R_a T) \ . \tag{3}$$

The MSE of the environment is given by h, q_v is the specific humidity of the environment, and R_a is the dry-air gas constant. The plume equations are integrated vertically by first specifying the temperature, total water, and pressure at plume base. If the plume is initially unsaturated, as it typically is when initialized with values taken from the near-surface level of a CRM simulation, equations 1–3 are advanced with the entrainment rate ϵ set to 0 to generate a dry adiabat until saturation occurs. At and above the level of plume saturation, the specific humidity and the MSE of the environment are calculated using the supplied RH profile and the known temperature of the plume/environment. The supplied entrainment profile is then used to calculate the plume's q_t and h^* at the next vertical step with a simple forward-difference method. A root-solver is used to calculate the temperature that is consistent with the known value of h^* at the next vertical step. Any water in excess of q_v^* is dumped into the q_c category. As it is written, equation 2 assumes no fallout of condensed water, but our plume model includes a parameter, γ, that determines what fraction of liquid water precipitates out at each step. If $1 \geq \gamma > 0$, any new q_c that forms when stepping vertically is reduced by the factor $(1-\gamma)$. This "precipitation" is removed at constant pressure and temperature. Since we neglect the effect of water on the density and pressure of air in this simple model, q_c only functions as a reserve of liquid water that can re-evaporate to maintain saturation after entrainment reduces the specific humidity of the plume. Iterating this procedure generates a plume/environment temperature profile. The buoyancy of an undilute parcel is then computed by lifting a parcel that conserves its (simple) MSE, computing its temperature as a function of height, and comparing to the plume/environment temperature profile. Note that the zero-buoyancy plume is always colder than the undilute parcel—the zero-buoyancy model does not predict an LNB. To get a value of CAPE from the buoyancy profile predicted by the zero-buoyancy model, one must supply an upper bound for the CAPE integration. The plume models are implemented in Python; code is available from the first author upon request.

Text S2: Complex zero-buoyancy model

Like the simple version, the complex zero-buoyancy model operates on the principle of neutrality between an entraining plume and its environment, but takes full account of the ice phase and the effects of water on the density and heat capacity of air. The total water mass fraction in this case is $q_t = q_v + q_l + q_s$, where q_l and q_s are the mass fractions of liquid and solid water, respectively. The MSE is given by

$$h = c_{pm}(T - T_0) + q_v(E_{0v} + R_v T_0) - q_s E_{0s} + gz, \tag{4}$$

where c_{pm} is the constant-pressure specific heat capacity of moist air, $T_0 = 273.16$ K is the triple-point temperature, E_{0v} is the difference in specific internal energy between water vapor and liquid at the triple-point temperature, R_v is the gas constant for water vapor, and E_{0s} is the difference in specific internal energy between water liquid and solid at the triple-point temperature. The moist-air heat capacity, c_{pm}, is given as a mass-fraction-weighted linear combination of the constant-pressure heat capacities of dry air (subscript a) and the three water phases (subscripts v, l, and s): $c_{pm} = q_a c_{pa} + q_v c_{pv} + q_l c_{pl} + q_s c_{ps}$.

The complex version of the zero-buoyancy model integrates the same plume equations as the simple version, but the pressure equation is replaced with

$$\partial_z \log p = \frac{-g}{R_m T_e} \ , \tag{5}$$

where $R_m = q_v R_v + (1 - q_v) R_d$ is the gas constant for moist environmental air and T_e is the environment temperature. The vertical integration of the plume equations is carried out as for the simple model, but since we include the effects of water phases on the density of air in this case, the neutrality of the entraining plume is enforced as a constraint on density rather than temperature. A rootsolving algorithm uses the known plume density and the supplied environmental relative humidity to calculate the environmental temperature that is consistent with a neutrally buoyant plume. The complex version of the zero-buoyancy model also includes the full effects of the ice phase; q_v^* is defined with respect to liquid at temperatures warmer than the triple-point temperature (273.16 K), with respect to ice at temperatures below 240 K, and as a linear combination of the two at temperatures in between. This corresponds to a non-isothermal mixed phase regime between the triple-point temperature and the temperature of homogeneous freezing; the partitioning of condensates in the plume transitions linearly in temperature from all-liquid to all-ice between these two temperatures. (For a more complete description of the moist thermodynamics used in this model, including explicit equations for q_v^*, see the appendix of *Romps* (2015)). To calculate undilute parcel buoyancy, a near-surface parcel is lifted assuming conservation of MSE - CAPE, and this parcel's density is then compared to the plume/environment density profile produced by the zero-buoyancy model. The plume models are implemented in Python; code is available from the first author upon request.

Text S3: Method for nudging RH

There are many ways one could "nudge" relative humidity in a numerical model, so here we will be explicit about how this forcing was implemented in our simulations. Our relative humidity nudging was performed by nudging the local q_v according to

$$F_{q_v} = \rho \frac{\mathrm{RH}^\dagger(z) q_v^* - q_v}{\tau}, \tag{6}$$

where $\mathrm{RH}^\dagger(z)$ is the target RH profile and the nudging timescale is τ. Note that F_{q_v} has units of density per time, and represents an artificial convergence of pure water vapor into the Eulerian finite volumes in the numerical model (we use "convergence" as shorthand for convergence or divergence). That is, F_{q_v} appears as a source in the governing equation for water vapor as follows:

$$\partial_t \left(q_v \rho \right) = -\vec{\nabla} \cdot \left(q_v \rho \vec{u} \right) + e - \vec{\nabla} \cdot \vec{d_v} + F_{q_v}, \tag{7}$$

where the other source terms are the resolved-flow moisture convergence $-\vec{\nabla} \cdot (q_v \rho \vec{u})$, the evaporation e, and the convergence of diffusive vapor fluxes $-\vec{\nabla} \cdot \vec{d_v}$, which is nonzero only in the near-surface level in our model.

However, since we are interested in the effect of environmental relative humidity on the temperature profile of a convecting atmosphere, we need to adjust RH in such a way that the forcing itself has negligible effects on temperature. The convergence of water vapor does work on the gas in a finite volume, and therefore has an effect on temperature. To counteract this, we also specify a countervailing convergence of an equal and opposite number of moles of dry air per volume per time:

$$F_{q_a}(z) = -\frac{R_v}{R_a} F_{q_v}. \tag{8}$$

The end result of this combination of forcings is effectively a mole-for-mole swap of dry air and water vapor. The corresponding effect on the model's finite-volume energy budget was accounted for by keeping track of the enthalpies of the exchanged gases. We apply this relative humidity nudging in every model level below 15 km, but not in the stratosphere.

To minimize the possibility of convective preconditioning (i.e., the probability that a developing cloud will grow through the moist detritus of a prior convective event), we should like to use a short τ in the free troposphere. However, if we nudge RH locally on too short a timescale near cloud base, we will never give clouds a chance to be born. Therefore, we specify an altitude-dependent τ given by

$$\tau(z) = \begin{cases} 1 \text{ minute} & z < 200 \text{ m} \\ 1 \text{ day} & 200\text{m} \le z \le 600 \text{ m} \\ \frac{10^7}{z^{1.35}} \text{ seconds} & 600 \text{ m} < z \end{cases}. \tag{9}$$

This profile of τ has the desirable properties of maintaining constant humidity in the subcloud-layer, giving clouds a chance to become saturated in the neighborhood of cloud base (~500 m), and quickly adjusting the RH of air outside of clouds to the target value in the free troposphere.

References

Romps, D. M. (2015), MSE minus CAPE is the true conserved variable for an adiabatically lifted parcel, *Journal of the Atmospheric Sciences*, pp. 0–13.

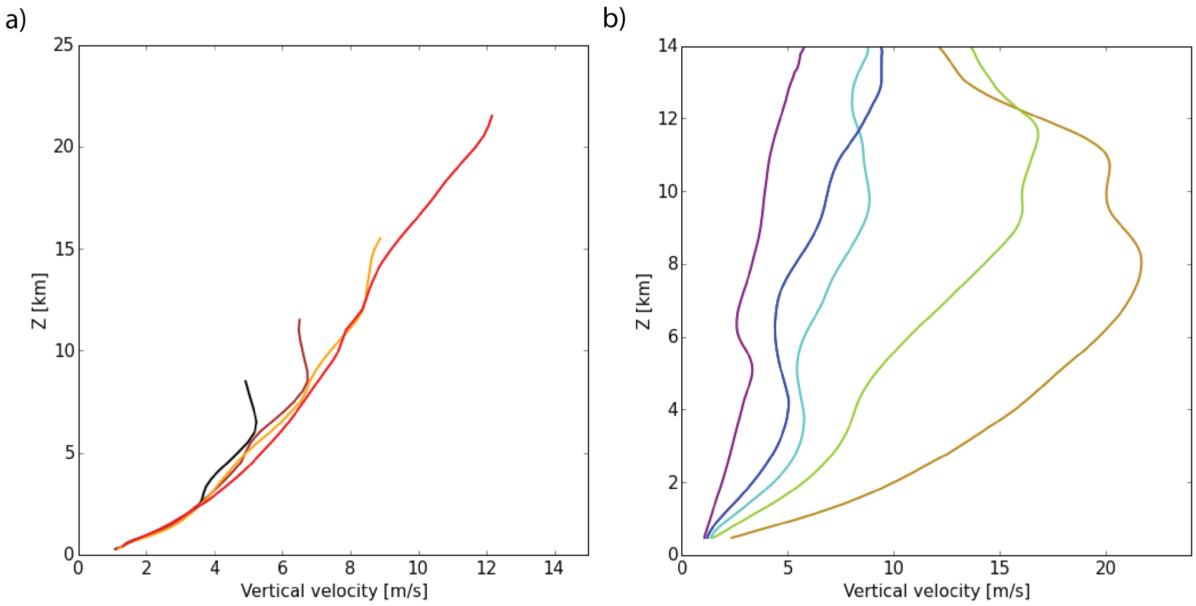

Figure S1. Profiles of mean vertical velocity in cloud updrafts in the CRM simulations from (a) the SST-warming experiment, and (b) the RH-varying experiment. Colors correspond to the set of SSTs and target free-tropospheric RH values as in Figures 2a and 4a of the main text. Note that (a) and (b) have different horizontal and vertical scales.